Reconstruction of Light Spectra from Multispectral Images

Frank Sippel
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Motivation

Shaw et al. "Hyperspectral imaging and quantitative analysis for prostate cancer detection"
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Shaw et al. "Hyperspectral imaging and quantitative analysis for prostate cancer detection"

Williams et al. "Classification of maize kernels using NIR hyperspectral imaging"
Problem Statement

Multispectral images

Imaging

Reconstruction

Pixel (250, 325)

Hyperspectral images
Imaging pipeline

\[ c_i = \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} q(\lambda) r(\lambda) f_i(\lambda) o(\lambda) m(\lambda) d\lambda \]

Often light source, imaged object, lens, and camera spectral transfer functions not known.

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Imaging pipeline

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Pipeline

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Imaging pipeline
Reconstructable spectrum: 
\[ s(\lambda) = q(\lambda)r(\lambda)o(\lambda)m(\lambda) \]
Mathematical Formulation

- Reconstructable spectrum: \( s(\lambda) = q(\lambda) r(\lambda) o(\lambda) m(\lambda) \)

- Substituted equation: \( c_i = \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} f_i(\lambda) s(\lambda) \, d\lambda \rightarrow \) Inverting integration cumbersome
Mathematical Formulation

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- **Substituted equation:** \( c_i = \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} f_i(\lambda)s(\lambda) \, d\lambda \rightarrow \text{Inverting integration cumbersome} \)

- **Discretization:** \( c_i = \sum_{j=1}^{N} F_{ij} s_j \) or \( c = Fs \)
  - \( c \in \mathcal{R}^M \): Multispectral channels for one pixel
  - \( F \in \mathcal{R}^{M \times N} \): Sampled filter as matrix
  - \( s \in \mathcal{R}^N \): Sampled spectrum for one pixel

**Underdetermined System**

- Typically: \( M \ll N \rightarrow \text{Underdetermined linear system of equations} \)
- Prior knowledge must be embedded
Mathematical Formulation

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Underdetermined System

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- Prior knowledge must be embedded
Spectra Statistics

- **Raw value**
  - Occurrences: $1e7$
  - Distribution:
    - X-axis: 0.0 to 1.0
    - Y-axis: 0.0 to 2.0

- **First order differences**
  - Occurrences: $1e7$
  - Distribution:
    - X-axis: -0.05 to 0.05
    - Y-axis: 0.0 to 4.0

- **Second order differences**
  - Occurrences: $1e7$
  - Distribution:
    - X-axis: -0.05 to 0.05
    - Y-axis: 0.0 to 3.0
Smoothed Pseudoinverse

Optimization problem:

\[ \hat{s}^{SP} = \arg\min_{s} \|Ds\|_2^2 \]

s.t. \[ c = Fs \]

First-order difference matrix:

\[
D_1 = \begin{pmatrix}
1 & -1 & 0 & \cdots & 0 \\
0 & 1 & -1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0 & \cdots & 0 & 1 & -1
\end{pmatrix}
\]

Closed-form solution with \( M = D^T D + \alpha I \):

\[ \hat{s}^{SP} = M^{-1} F^T (FM^{-1} F^T)^{-1} c \]
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Pratt et al. "Spectral estimation techniques for the spectral calibration of a color image scanner"
Smoothed Pseudoinverse

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Noise

Image noise often generated by low exposure time, e.g., video

Reference

Noisy image

Most important noise sources:
- Content-dependent shot noise (Poisson)
- Dark current noise (Gaussian)
- Amplifier noise (Gaussian)
- Reset noise (Gaussian)

Poisson noise cumbersome to model

Poisson distribution can be approximated by a Gaussian for higher means

Model: Additive white Gaussian noise ($n$): $c_F = f + n$
Noise

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Noisy image

\[ P(k; l) \]

0.0 0.1 0.2

0 20 40 60

\( l = 2 \)
\( l = 15 \)
\( l = 35 \)
\( l = 50 \)

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July 21, 2021
Page 7
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Noisy image

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P(k; l) = \begin{cases} 
0 & \text{for } l = 2 \\
0.1 & \text{for } l = 15 \\
0.2 & \text{for } l = 35 \\
0 & \text{for } l = 50 
\end{cases}
\]
Noise

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- Model: Additive white Gaussian noise ($\mathbf{n}$):
  \[
  \mathbf{c} = \mathbf{F}s + \mathbf{n}
  \]
Wiener Filter

Reminder: $c = Fs + n$

Covariances (independence between noise and spectrum, zero mean):

- Hyperspectral covariance:
  \[ K_r = E\{ss^T\} \]

- Multispectral covariance:
  \[ K_c = E\{Fss^TF^T\} + E\{nn^T\} = FK_rF^T + N \]

- Multispectral-hyperspectral cross-covariance:
  \[ K_{rc} = K_rF^T \]

Wiener filter:
\[ \hat{s}_{WF} = K_{rc}K_c^{-1}c = K_rF^T(FK_rF^T + N)^{-1}c. \]

Smoothed pseudoinverse:
\[ \hat{s}_{SP} = M^{-1}F^T(FM^{-1}F^T)^{-1}c \]

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Pratt et al. "Spectral estimation techniques for the spectral calibration of a color image scanner"
Spatio-spectral Wiener Filter

- Idea: Exploit spatial correlation to encounter noise

\[ \hat{S}_{SSW}(x,y) = P \hat{F}^T (\hat{F} \hat{F}^T + \hat{N})^{-1} C_{x,y} \]

- \( \hat{F} \) and \( \hat{N} \) extended (block-)diagonal matrices
- \( C_{x,y} \): Vectorized image block
- \( P \): Picks out spectrum of pixel in the center
- Combine spatial and spectral covariance: \( K = K_s \otimes K_r \).

\( \otimes \): Kronecker product

Murakami et al. "Color reproduction from low-SNR multispectral images using spatio-spectral Wiener estimation"
Spatio-spectral Wiener Filter

▶ Idea: Exploit spatial correlation to encounter noise

▶ Adjust correlation matrix:

\[ \hat{S}^{SSW}(x, y) = PK\hat{F}^T(\hat{FK}\hat{F}^T + \hat{N})^{-1}C_{b,x,y} \]

▶ \(\hat{F}\) and \(\hat{N}\) extended (block-)diagonal matrices

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Spatio-spectral Wiener Filter

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Spatio-spectral Wiener Filter

- Spatial correlation modeled by first order Markov process:
  \[ K_s = R(p) \otimes R(p) \]

- with (blocksize \( B_r \), decay \( p \)):
  \[
  R(p) = \begin{pmatrix}
  p^0 & p^1 & \cdots & p^{B_r-1} \\
  p^1 & p^0 & \cdots & p^{B_r-2} \\
  \vdots & \vdots & \ddots & \vdots \\
  p^{B_r-1} & p^{B_r-2} & \cdots & p^0
  \end{pmatrix}
  \]

- \( R(p) \otimes R(p) \) for vertical and horizontal direction

Murakami et al. "Color reproduction from low-SNR multispectral images using spatio-spectral Wiener estimation"
Novel Reconstruction Method

- Idea: Generate guide image from multispectral images
- Independence of noise leads to much less noise in guide image
Structure-preserving Reflectance Estimation

Guide is weighted average of multispectral images:

\[ G(x, y) = \sum_{i=1}^{M} w_i \tilde{C}_i(x, y) + \sum_{i=1}^{M} w_i n_i = \]

\[ = w^T \tilde{C}(x, y) + w^T n \]

Sippel et al. "Structure-preserving spectral reflectance estimation using guided filtering "
Guide is weighted average of multispectral images:

\[
G(x, y) = \sum_{i=1}^{M} w_i \hat{C}_i(x, y) + \sum_{i=1}^{M} w_i n_i = w^T \hat{C}(x, y) + w^T n
\]

Optimize SNR:

\[
\arg\max_w \frac{\mathcal{E} [w^T \hat{C}(x, y) \hat{C}(x, y)^T w]}{\mathcal{E} [w^T nn^T w]}
\]

Sippel et al. "Structure-preserving spectral reflectance estimation using guided filtering"
Guided filtering idea: Guide indicates regions with edges and smooth regions.
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Assumption: Noiseless image \( \mathbf{S}_i(u, v) \) is linear combination of \( \mathbf{G} \):

\[
\mathbf{S}_i(u, v) = a_{x,y} \mathbf{G}(u, v) + b_{x,y} \quad \forall (u, v) \in w_{x,y}
\]

Usually true in small window \( w_{x,y} \)
Guided filtering idea: Guide indicates regions with edges and smooth regions

Assumption: Noiseless image $\hat{S}_i(u, v)$ is linear combination of $G$:

$$\hat{S}_i(u, v) = a_{x,y}G(u, v) + b_{x,y} \quad \forall (u, v) \in w_{x,y}$$

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Average in smooth regions via $b_{x,y}$
Structure-preserving Reflectance Estimation

- Guided filtering idea: Guide indicates regions with edges and smooth regions

- Assumption: Noiseless image $\tilde{S}_i(u, v)$ is linear combination of $G$:

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- Usually true in small window $w_{x,y}$

- Average in smooth regions via $b_{x,y}$

- Use linear regression in regions with edges via $a_{x,y}$
Evaluation

- Images from natural scenes
- Wavelengths: 440nm to 920nm
- 10nm steps → 49 hyperspectral channels

Eckhard et al. "Outdoor scene reflectance measurements using a Bragg-grating-based hyperspectral imager"
Evaluation

▶ Images from natural scenes

▶ Wavelengths: 440nm to 920nm

▶ 10nm steps → 49 hyperspectral channels

▶ Filters: Different colors indicate different filters

Eckhard et al. "Outdoor scene reflectance measurements using a Bragg-grating-based hyperspectral imager"
Evaluation

- Metric spectral angle:

\[ SA(s, \hat{s}) = \arccos\left( \frac{s^T}{\|s\|_2} \frac{\hat{s}}{\|\hat{s}\|_2} \right) \]

- Measures the angle between two spectrum vectors
- Lower is better

Simulated Poisson noise to evaluate noise behaviour:

\[ C_i(x,y) \sim P(l_iC_i(x,y)) \]

- Intensity level \( l \) indicates lighting conditions
- Low intensity level corresponds to high noise influence
Evaluation

- Metric spectral angle:

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Simulated poisson noise to evaluate noise behaviour:

\[
C_i(x,y) \sim \mathcal{P}(l_\text{\hat{C}_i}(x,y))
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- Simulated poisson noise to evaluate noise behaviour:

\[ C_i(x, y) \sim \mathcal{P} \left( \frac{l \tilde{C}_i(x, y)}{l} \right) \]
Evaluation

▶ Metric spectral angle:

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Spectroscopy has tons of applications
Conclusion

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- Multispectral camera to record scene → Cheaper than a hyperspectral camera
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▶ Basic methods: Smoothed pseudoinverse, Wiener filter
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- Goal: Reconstruct spectra from multispectral images
- Basic methods: Smoothed pseudoinverse, Wiener filter
- Spatial correlation to encounter noise
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- Novel reconstruction method for noisy scenarios based on guided filtering